Beam Design
Beam Design

- Beam Design
- Stress
- Axial Stress
- Strain
- Factor of Safety
- Bending Stress
- Shear Stress
- Beam Selection

- Deflection
- Evaluation and Redesign
Beam Design

• Beams are designed to safely support the design loads.
• Beams are primarily designed for bending and shear.
• Beam deflection must be checked.
• Beams are sized to minimize material.
Steps in Beam Design

1. Establish the design loads
2. Analyze the beam

3. Select the preliminary member
4. Evaluate the preliminary design
5. Redesign (if needed) – Repeat the above steps as necessary to achieve a safe and efficient design

6. Design and detail the structural component
Stress

- A measure of the magnitude of the internal forces acting between particles of the member resulting from external forces
- Expressed as the average force per unit area

\[
\text{Stress} = \frac{\text{Force} \ F}{\text{Area} \ A}
\]
Axial Stress

- Tension and compression
- Axial stress represented by

\[ \sigma = \frac{F}{A} \]

Where $F$ = applied force
A = cross sectional area resisting load
Axial Stress

**Example:** Find the tensile stress of a 2 in. \( \times \) 3 in. tension member subjected to a 45 kip axial load.

\[
\sigma = \frac{F}{A} = \frac{45 \text{ k}}{(2 \text{ in})(3 \text{ in})} = 7.5 \text{ ksi} = 7500 \text{ psi}
\]
Strain

- The change in size or shape of a material caused by the application of external forces
- Axial strain,

\[ \varepsilon = \frac{\Delta L}{L} \]

Where \( \Delta L \) = change in length
\( L \) = original length
Strain

**Example**: Calculate the strain if a 16 ft long structural member elongates 1.5 in. when subjected to a tensile load of 67 kips.

\[
\varepsilon = \frac{\Delta L}{L} = \frac{1.5 \text{ in}}{(16 \text{ ft})(\frac{12 \text{ in}}{1 \text{ ft}})} = 0.0078
\]
Stress and Strain

• In many materials, stress is directly related to strain up to a certain point.
Stress and Strain

- Elastic behavior – material will return to its original shape when unloaded
- Plastic behavior – material will retain some deformation when unloaded

- Structural members are designed to act elastically during the service life of a building
Factor of Safety

• The ratio of the maximum safe load to the maximum allowable design load
• Magnitude of the factor of safety varies depending on the loading conditions and type of forces induced
Allowable Strength Design (ASD)

• Strength is related to stress
  – Strength indicates internal force
  – Stress indicates internal force per unit area

• ASD limits the maximum internal force within a structural member

• Maximum safe load = nominal strength
  – Internal force that causes yielding across the entire cross section

• Maximum allowable load = allowable strength
Allowable Strength Design (ASD)

\[ \Omega \leq \frac{F_n}{F_a} \quad \text{OR} \quad F_a \leq \frac{F_n}{\Omega} \]

Where \( \Omega \) = factor of safety
\( F_n \) = nominal strength
\( F_a \) = internal force due to design loads
\( \frac{F_n}{\Omega} \) = allowable strength
Allowable Bending Strength

\[ M_a \leq \frac{M_n}{\Omega_b} \quad \text{OR} \quad M_n \geq M_a \Omega_b \]

Where

- \( \Omega_b = 1.67 \) = factor of safety for bending
- \( M_n \) = nominal bending moment strength
- \( M_a \) = internal bending moment due to design loads
- \( \frac{M_n}{\Omega_b} \) = allowable bending strength
Nominal Bending Strength

\[ M_n = F_y Z_x \]

Where

- \( F_y \) = yield stress of steel
- \( Z_x \) = plastic section modulus

NOTE: We will assume that every beam and girder is laterally supported along its length so that it will not buckle under loading. If a beam is not laterally supported, buckling must be checked.
Plastic Section Modulus, $Z$

- Section property
- Indicates the moment carrying capacity of a member
- Available in tabular form in design manuals

![Table](http://www.structural-drafting-net-expert.com/)
Plastic Section Modulus, $Z$

- I-shaped members are excellent choices for beams
- Structural steel wide flange
  - Designation W
  - Example: W12 x 58
- Stronger when bending about the $x - x$ axis, $Z_x$
Allowable Shear Strength

\[ V_a \leq \frac{V_n}{\Omega_v} \quad \text{OR} \quad V_n \geq V_a \Omega_v \]

Where \( \Omega_v = 1.5 \) = factor of safety for shear
\( V_n \) = nominal shear strength
\( V_a \) = internal shear force due to design loads
\( \frac{V_n}{\Omega_v} \) = allowable shear strength
Nominal Shear Strength

\[ V_n = 0.6 F_y A_w \]

Where

\[ V_n = \text{nominal shear strength} \]
\[ F_y = \text{yield stress of steel} \]
\[ A_w = \text{area of the web} \]
\[ A_w = d \cdot t_w \]
Beam Deflection

• Deflection limit supporting plaster ceilings
  – L/240 for Dead + Live Loads
  – L/360 of Live Load

• Deflection limit supporting non-plaster ceilings
  – L/180 for Dead + Live Loads
  – L/240 of Live Load

• WHY?
  – Ceiling cracks in plaster
  – Roof ponding (flat roofs)
  – Visual or psychological reasons
  – Designer’s judgment
Beam Selection Process

- Select beam based on bending moment
- Check shear strength
- Check deflection
- Revise beam selection as necessary
Choose the lightest wide flange steel section available to support a **live load of 790 plf** and a **dead load of 300 plf** over a simple span of 18 feet. Assume the beam will support a plaster ceiling. Use $F_y = 50$ ksi.
Beam Design Example

Max Shear: 9,810.00
Max Bending Moment: 44,145.00
Bending Strength

Select beam based on bending.

\[ M_n \geq M_a \Omega_b \]

where \( M_a = 44,145 \text{ ft} \cdot \text{lb} \)

\( \Omega_b = 1.67 \)

\[ M_n \geq (44,145 \text{ ft} \cdot \text{lb})(1.67) \]

\[ M_n \geq 73,722 \text{ ft} \cdot \text{lb} \]
Bending Strength

Since \( M_n = F_y Z_x \)

\[
F_y Z_x \geq 73,722 \text{ ft} \cdot \text{lb}
\]

\[
Z_x \geq \frac{73,722 \text{ ft} \cdot \text{lb}}{F_y}
\]

\[
Z_x \geq \frac{73,722 \text{ lb}}{50,000 \text{ lb/in}^2}
\]

\[
Z_x \geq 177 \text{ in}^3
\]

W10 x 17 works \( Z_x = 18.7 \text{ in}^3 > 17.7 \text{ in}^3 \)

But a W 12 x 16 weighs less with \( Z_x = 20.1 \text{ in}^3 > 17.7 \text{ in}^3 \)
Shear Strength

Check shear strength.

\[ V_n \geq V_a \Omega_v \]

where

\[ V_a = 9,810 \, lb \]

\[ \Omega_v = 1.5 \]

\[ V_n \geq (9810 \, lb)(1.5) \]

\[ V_n \geq 14,715 \, lb \]
Shear Strength

Since \[ V_n = 0.6F_yA_w \]
\[ V_n = 0.6F_y \cdot d \cdot t_w \]

For a \( W \, 12 \times 16 \)
\[ d = 11.99 \text{ in.} \quad t_w = 0.220 \text{ in.} \]

\[ V_n = 0.6 \left( 50,000 \frac{lb}{in^2} \right) \left( 11.99 \text{ in.} \right) \left( 0.220 \text{ in.} \right) \]

\[ V_n = 79,134 \text{ lb} \]
\[ 79,134 \text{ lb} \geq 14,715 \text{ lb} \]

\( W\, 12 \times 16 \) works
Check Deflection

Deflection limit supporting plaster ceilings

– L/240 for Dead + Live Loads

\[
\frac{L}{240} = \frac{(18 \text{ ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)}{240} = 0.9 \text{ in}
\]

– L/360 of Live Load

\[
\frac{L}{360} = \frac{(18 \text{ ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)}{360} = 0.6 \text{ in}
\]
Check Deflection

Maximum deflection due to design loads

\[ \Delta_{\text{max}} = \frac{5wl^4}{384EI} \]

– Dead + Live Load Deflection

\[ \Delta_{\text{max}} = \frac{5 \left( 1090 \frac{lb}{ft} \cdot \frac{1 ft}{12 \text{ in}} \right) \left( 18 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \right)^4}{384(29,000,000 \frac{lb}{in^2})(103 \text{ in}^4)} \]

= 0.87 in < 0.90 in

W 12 x 16 will work
**Check Deflection**

Maximum deflection due to design loads

\[ \Delta_{\text{max}} = \frac{5wl^4}{384EI} \]

- **Live Load Deflection**

\[ \Delta_{\text{max}} = \frac{5 \left( \frac{790 \text{lb}}{\text{ft}} \cdot \frac{1 \text{ft}}{12 \text{ in}} \right) \left( 18 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \right)^4}{384 \left( 29,000,000 \text{ lb/in}^2 \right) \left( 103 \text{ in}^4 \right)} \]

\[ = 0.63 \text{ in} < 0.60 \text{ in} \]

W 12 x 16 will not work
Evaluation and Redesign
Check Deflection

Try W 12 x 19

\[ I_x = 130 \text{ in}^4 \quad Z_x = 24.7 \text{ in}^3 \quad d = 12.16 \text{ in} \quad t_w = 0.235 \text{ in} \]

Dead + Live Deflection

\[ \Delta_{max} = \frac{5 \left( 790 \frac{lb}{ft} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \right) \left( 18 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \right)^4}{384 \left( 29,000,000 \frac{lb}{in^2} \right) \left( 130 \text{ in}^4 \right)} \]

\[ = 0.50 \text{ in} < 0.60 \text{ in} \]

W 12 x 19 OK
Evaluation and Redesign
Check Shear and Bending Moment

By inspection, the plastic section modulus and web area for the W12 x 19 are larger than those for the W12 x 16 and are therefore sufficient to safely support the bending moment and shear.

Use a W12 X 19
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