We require from buildings two kinds of goodness: first, the doing their practical duty well: then that they be graceful and pleasing in doing it.

- John Ruskin
Beam

• Structural member that carries a load that is applied transverse to its length
• Used in floors and roofs
• May be called floor joists, stringers, floor beams, or girders
Beam Analysis

Beams are designed to carry the Shear and Bending Moment caused by the design loads.
Chasing the Load

• The loads are initially applied to a building surface (floor or roof).
• Loads are transferred to beams which transfer the load to another building component.
Static Equilibrium

• The state of an object in which the forces counteract each other so that the object remains stationary

• A beam must be in static equilibrium to successfully carry loads
Static Equilibrium

• The loads applied to the beam (from the roof or floor) must be resisted by forces from the beam supports.
• The resisting forces are called reaction forces.
Reaction Forces

• Reaction forces can be linear or rotational.
  – A linear reaction is often called a shear reaction (F or R).
  – A rotational reaction is often called a moment reaction (M).

• The reaction forces must balance the applied forces.
Beam Supports

The method of support dictates the types of reaction forces from the supporting members.

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<td>![Roller Diagram]</td>
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Beam Types

Simple

Continuous

Cantilever

Moment

(fixed at one end)
Beam Types

Fixed

Moments at each end

Propped – Fixed at one end; supported at other

Overhang
Note: When there is no applied horizontal load, you may ignore the horizontal reaction at the pinned connection.
Fundamental Principles of Equilibrium

\[ \sum F_y = 0 \]  
The sum of all vertical forces acting on a body must equal zero.

\[ \sum F_x = 0 \]  
The sum of all horizontal forces acting on a body must equal zero.

\[ \sum M_0 = 0 \]  
The sum of all moments (about any point) acting on a body must equal zero.
Moment

• A moment is created when a force tends to rotate an object.
• The magnitude of the moment is equal to the force times the perpendicular distance to the force (moment arm).

\[ \text{Moment} = F \cdot d \]

\[ \text{Moment arm} \]

\[ F \cdot \perp = M \]
Calculating Reaction Forces

Sketch a beam diagram.
Calculating Reaction Forces

Sketch a free body diagram.
Calculating Reaction Forces

Use the equilibrium equations to find the magnitude of the reaction forces.

– **Horizontal Forces**
– Assume to the right is positive

\[
\sum F_x = 0
\]

\[
F_{xA} = 0
\]
Calculating Reaction Forces

- **Vertical Forces**
- Assume up is positive \( + \uparrow \)

\[
\sum F_y = F_{yA} + F_{yB} - 4000\text{lb} - \left( \frac{650\text{lb}}{20\text{ft}} \right) = 0
\]

\[
F_{yA} + F_{yB} = 4000\text{lb} + 13,000\text{lb}
\]

\[
F_{yA} + F_{yB} = 17,000\text{lb}
\]
Calculating Reaction Forces

- **Moments**
- Assume counter clockwise rotation is positive

\[ \sum M_A = 0 \]

\[ (F_{yB})_A + (F_{yB})_B = 0 \]

\[ (20)(14)F_{yB} + (20)(13)F_{yB} = 0 \]

\[ 20F_{yB}(13000 - 1540000) = 0 \]

\[ F_{yB} = \frac{1540000}{20 ft} \]

\[ F_{yB} = 7700 \text{ lb} \]
Calculating Reaction Forces

• Now that we know \( F_{yB} \), we can use the previous equation to find \( F_{yA} \).

\[
F_{yA} + F_{yB} = 17,000 \text{ lb}
\]

\[
F_{yA} + 7700 \text{ lb} = 17,000 \text{ lb}
\]

\[
F_{yA} = 9300 \text{ lb}
\]
Shear Diagram

0 = \( F_{xA} \)

9300 lb = \( F_{yA} \)

\( F_{yB} = 7700 \text{ lb} \)

\( P = 4000 \text{ lb} \)

\( w = 650 \frac{\text{lb}}{ft} \)

\( \int \left( 650 \frac{\text{lb}}{ft} \right) (6') = 3900 \text{ lb} \)

\( 4000 \text{ lb} \)

\( 1400 \text{ lb} \)

\( -7700 \text{ lb} \)
Moment Diagram

Kink in moment curve

\[ \int (650 \frac{\text{lb}}{\text{ft}}) \times 16' = 3900 \text{lb} \]

\[ -1400 \text{ lb} \]

\[ \frac{1400 \text{ lb}}{650 \text{ lb/ft}} = 215 \]
Moment Diagram

\[ \sum M_p = 0 \]

\[ M + (4000 \text{ lb})(2.15 \text{ ft}) = 9300 \text{ lb} \times 2.15' \]

\[ M = \frac{4000 \times 2.15}{2} - 9300 \times 2.15' \]

\[ M = 45608 \text{ ft lb} \]

\[ M_{\text{max}} = 45608 \text{ ft lb} \]
Moment Diagram

\[ M_{\text{max}} = 45,608 \text{ ft} \cdot \text{lb} \]
Moment Diagram

\[ M_{\text{max}} = \text{Area A} + \text{Area B} + \text{Area C} \]
\[ = \frac{1}{2}(6 \text{ ft})(3900 \text{ lb}) + (6 \text{ ft})(5400 \text{ lb}) + \frac{1}{2}(2.15 \text{ ft})(1400 \text{ lb}) \]
\[ = 45,605 \text{ ft} \cdot \text{lb} \]
Beam Analysis

- Example: simple beam with a uniform load, $w_1 = 1090 \text{ lb/ft}$
- Span = 18 feet

Test your understanding: Draw the shear and moment diagrams for this beam and loading condition.
Shear and Moment Diagrams

Max. Moment = 44,145 l ft-lb        Max. Shear = 9,810 lb